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Book Review Critique de livre

“Group Theory in a Nutshell for Physicists”, by A. Zee, Princeton University Press, 2016, ISBN: 9780691162690, 632 pages, Price: 125.95$

After superb books on Quantum Field Theory and on General Relativity, Anthony Zee came out with a new textbook on another subject of high importance in physics: Group Theory. This is a mathematical subject that has become an essential tool in modern physics since the last century and many physics students unfortunately graduate without having been exposed to a proper introduction to it. Many good books are already available on the subject but Zee’s “Group Theory in a Nutshell for Physicists” is different from most of them in many aspects. The great clarity of the explanations, the variety of topics covered and the emphasis put on the physics grant it in my opinion the title of “Group theory book that every physicist should read”.

The book starts with a review of linear algebra to set the notation and put every reader on the same page, especially the beginners. Chapter 1 introduces the notion of group with multiple examples of discrete and continuous ones. Chapter 2 presents what representations of groups are along with their various characteristics. This is where the most important properties of character tables of discrete groups are discussed and explained with many examples. Chapter 3 very quickly discusses the relation between degeneracies in quantum mechanics and group theory. Chapter 4 is by far the most important one because it explains how to build irreducible representations of the orthogonal and unitary groups and algebras with the tensor method and the ladder operators method. It shows how to multiply these representations together as well and spends a good amount of time on the relation between SO(3) and SU(2). Chapter 5 uses the notions developed previously to talk about isospin and the eightfold way in particle physics. Chapter 6 goes over the Killing-Cartan classification of Lie algebras using roots of algebras and Dynkin diagrams. This one is not really related to the rest of the book and has nothing to do with physics so it can easily be omitted. Chapter 7 is about the Lorentz algebra in relativistic physics, more specifically the spinor representations of SO(N), leading to the Weyl and Dirac spinors in Quantum Field Theory. Chapter 8 goes quickly over some specific aspects of group theory, like the conformal group and the group theory behind the expansion of the universe. Finally chapter 9 shows how grand unification using SU(5) and SO(10) work, after briefly introducing the Standard Model of particle physics. This summary shows clearly that this book covers a lot of material and in great detail. I however felt that the discrete group part of the book could have been bigger.

Like in all of his nutshell books, Zee starts his discussion with the very basic notions and builds towards really advanced topics. Group theory as a mathematical subject doesn’t require a deep background so any reader who understands the linear algebra review can understand all of the powerful mathematics developed in the text. Furthermore, every physicist who has some knowledge of quantum mechanics can understand almost all of the book. Of course the applications to more advanced topics like QFT and the Standard Model can be intimidating to

people who don’t know anything about the subjects, but the group theoretic applications that are discussed require only a minimal knowledge that Zee explains very clearly. In addition, there is a good alternation between mathematics and physics in the ordering of the topics. As soon as there is enough maths known to talk about a physical application, it is discussed. This makes the more physically minded reader not lose interest in the reading and it helps make a clear connection between the group theory and the physics. The book is also separated such that the essential group theory knowledge that everyone should know is presented before the advanced applications and someone uninterested in these can stop reading there, approximately after chapter 5.

From the point of view of a graduate student who is more familiar with the physics discussed, it is very interesting to see a different perspective on material that is already known so this is where the physical applications make the most sense. Of course this could be said for any good group theory book for physicists, but what sets this one apart from the others is the special topics that are covered, mostly in the appendices at the end of the chapters and in the interludes. Indeed, it is not very common to introduce group manifolds or discuss the conformal algebra in that kind of book. I personally really enjoyed reading about the classification of Lie algebras and the derivation of spinor representations in general dimensions. The section on the Lorentz group should in my opinion be read by every QFT student because it is clearer than what is done in most QFT textbook.

Zee’s style of writing has always been one of the things that make his textbooks so popular. He writes as if he was giving a lecture to a class and not like if he was writing a book. You know that he thought about the subject a lot and he is not scared of asking through the character known as Confusio questions that many people would qualify as “stupid”. It is also obvious in the text that it is written by a physicist for physicists. The author mentions from the get go in the preface that one should not expect a lot of mathematical rigor but more intuition and examples. This of course makes the reading more fluid and thus more enjoyable for a physicist.

The problems at the end of each section are a good mix of exercises that help understand the material and others that introduce new notions. However, towards the end of the book there starts to be fewer problems and it would have been nice to have more. An interesting idea would have been to include longer problems to go deeper into the advanced topics that were covered quickly. Of course reading this book is a great exercise by itself since the best way to learn, as emphasized by the author many times, is to work out all the calculations that are left out in the text. One can’t get a full understanding of any topic in mathematics or physics without doing that, and this is especially true for group theory. Zee even leaves some results to prove by the reader with some hints just to make sure that they stop and think while reading.

To summarize, both new and advanced physicists will enjoy this book but in different ways. The new ones will learn all of the important mathematical notions from a very clear author with many examples but will maybe need to skip some of the physical applications to subjects that they are not familiar with. Advanced readers will enjoy learning about numerous topics that are included and are not generally covered in this context. Unlike the other nutshell books by Zee, this one contains enough to teach most readers everything they need to know about the

subject without having to study it more specifically elsewhere. I would (and already have) therefore recommend this book to literally every physics student I know to make sure that they know about this amazing subject that is group theory.

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