



**Differential Geometry and Lie Groups for Physicists**, *Marian Fecko*, Cambridge University Press, 2006, pp. 697, ISBN 0-521-84507-6, \$75.00

If I were to describe this book in one word, that word would have to be "different". There are many excellent differential geometry texts available, but this one stands out in many different ways. It's not a dry collection of abstract definitions and page-long proofs, as the pure mathematical texts tend to be. It's not an "excerpts from" or "crash course in" book, as the "for physicists" books often are. It is not a collection of lecture notes, like the Isham's "Modern Differential Geometry for Physicists". Marián Fecko deftly guides you through the material step-by-step, with all the rigour, but without the pain. When going through the chapters, definition by definition, proof by proof and hint by hint, you get an impression of a caring, experienced (and often quirkily funny, but never boring) tutor who really, really wants you to succeed. If one set out to write an interactive math book using a Socratic method of teaching (a fancy name for asking illuminating questions instead of giving ready answers), Fecko's book is how one might want to approach this. The author often invites you to "draw a picture", or "contemplate this until it is clear". Of course, the book has a fair number of visualizations already.

There are no problems at the end of each chapter, but that's because by the time you reached the end of the chapter, you feel like you've done your homework already, proving or solving every little numbered exercise, of which there can be between one and half a dozen per page. Fortunately, each chapter ends with a summary and a list of relevant equations, with references back to the text.

The book fulfills the "for physicists" part of the title not only in its approach, but also in the content. Chapters 14 and 18 cover Hamiltonian mechanics, chapter 16 describes the classical field theories (including electromagnetism and gravity), chapter 21 addresses gauge fields and chapter 22 talks introduces spinors and the Dirac operator using the Clifford algebra approach.

The material covered in the book is fairly standard for modern differential geometry texts. The first four chapters cover manifolds, vector fields, the metric, Lie derivatives and the symmetries. To make the Stokes theorem and its applications obvious, the author goes through the calculus of exterior forms (chapters 5 to 8). At this point the reader is ready for a taste of algebraic topology, in particular the cohomology theory using de Rham complex (chapter 9). Next four chapters introduce and explore in some detail the properties of Lie groups and Lie algebras. Chapters 14, 15, 16 and 18 are essentially the physics applications. The concept of bundles is explored in chapters 17 (tangent bundle), 19 (linear bundle) and 20 (principal G-bundle). Appendix A serves as a reference for the relevant algebraic structures, and even touches upon the category theory.

A somewhat idiosyncratic flavour of this text is reflected in the numbering: there are no numbered equations, it's the exercises that are numbered, and referred to later.

When working through the book I occasionally wished for a physical example or two, but this may be too much to ask for what is essentially a mathematical text. The rich structure of semi-Riemannian geometry, leading to the many surprising results in General Relativity (GR), is covered only briefly in chapter 16, so this book is not a substitute for any of the GR texts. Then again, at nearly 700 pages the book is already very hefty, and the mathematical foundation it provides should be enough to greatly simplify the task of learning GR properly.

A review of this book would be incomplete without an example of the author's humour, this one from the chapter on tangent spaces and vectors: "If we would like to visualize the concept of a vector in the sense of an equivalence class of tangent curves, we should assign something like a "bunch" or a "sheaf" of curves, all of them firmly bound together at the point P. And a good old arrow, which cannot be thought of apart from the vector, could be put at P in the direction of this bunch, too (so that it does not feel sick at heart that it had been forgotten because of some dubious novelties)." I'll leave it to the reader to decide whether it is funny or not.

The book is suitable for (and indeed arose from, as most such books do) an upper undergraduate or a graduate-level one- or two- (or three-, the book has plenty of material) term course in differential geometry, Lie groups and representations for Physics students. It may also be suitable for self-study, if you are tenacious enough.

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